It is important to be cautious when making conjectures. They may or may not be true. To demonstrate that a conjecture is false, you need to describe only one counterexample, which is an example for which the conjecture is false.

Many years ago, people thought the Earth was flat. Whenever a ship left on a voyage of exploration and never returned, it was believed that the ship had fallen off the edge of the Earth. The flat-Earth conjecture was believed by many until Ferdinand Magellan's ship arrived back in Portugal on September 6, 1522, after taking three years to sail around the world. This trip was seen as a counterexample to the conjecture that the Earth was flat.

Explore: Look for a Pattern

When 2 points are placed on the circumference of a circle and joined, 2 regions are formed.

When 3 points are placed on the circle and each point is joined to every other point, 4 regions are formed.

When 4 points are used, 8 regions are formed.

Make a conjecture by describing the pattern in the number of regions formed.

Inquire

1. Use the pattern to predict the number of regions formed when 5 points are used. Use a diagram to check your prediction. Is the pattern valid for 5 points?

2. Use the pattern to predict the number of regions formed when 6 points are used. Use a diagram to check your answer. Is the pattern valid for 6 points?

3. What conclusion can you make about your conjecture for describing the pattern in the number of regions formed?

4. Give a counterexample to show that each of the following conjectures is false:
   a. If you live in a country bordering the United States, then you live in Canada.

   b. If a quadrilateral has four right angles, then it is a square.

   c. A heavier-than-air mechanically driven vehicle that flies is an airplane.

5. Consider the conjecture: In any isosceles triangle, all three angles are acute.
   a. Sketch two examples of triangles for which the conjecture holds. (Label the angles.)
   b. Sketch one counterexample that shows the conjecture is not true. (Label the angles.)

Logic Power

Assume that no small cubes are missing from the back of the stack in the image below.
1. How many small cubes are needed to complete the large cube?
2. The faces on the outside of the large cube must all be red. The faces hidden inside the large cube must all be yellow. Some of the small cubes you found in question 1 need to have 2 red faces and 4 yellow faces.
   a. What other combinations of red and yellow faces are needed?
   b. Find how many small cubes are needed with each combination of coloured faces.

Part A: Practice

Give one example that supports each conjecture, and then give one counterexample that shows the conjecture is false.

Conjecture	Example	Counterexample
1. Provinces in Canada share a land border with the United States		
2. If the name of a province in Canada contains the letter s, then the province has a coastline.		
3. Canadian provincial capitals are south of the 53°N parallel of latitude.		
4. The square root of a number is smaller than the number.		
5. If the x-coordinate of a point in the Cartesian plane is positive, then the point is in the first quadrant.		
6. The square of a number is greater than the number.		
7. All multiples of 4 are divisible by 8.		
8. The square root of a number is a rational number.		

Draw a diagram to illustrate each of the following conjectures. Then draw a counterexample diagram showing the conjecture is not true.

Conjecture	Example	Counterexample
9. If a quadrilateral has two equal diagonals, then it is a rectangle.		
10. If two opposite angles in a quadrilateral are equal, then the other two opposite angles are supplementary.		
11. If a quadrilateral has four equal sides, then it is a square.		
12. If a triangle has exactly two obtuse exterior angles, then it is a right triangle.		
13. If the measures of two sides of a right triangle are 3 cm and 4 cm, then the third side measures 5 cm.		

Part B: Applications and Problem Solving

Use examples or counterexamples to confirm or deny each conjecture in questions 14-27.

14. If a scalene triangle has a 60° angle, then this is opposite the shortest side of the triangle.
15. From a point not on a line, it is possible to draw two perpendiculars to the line.
16. If a transversal is perpendicular to one of two parallel lines, then it is perpendicular to the other line.
17. If a quadrilateral has perpendicular diagonals, then it is a square.
18. A quadrilateral cannot have exactly three interior angles that are right angles.
19. If two interior angles of a quadrilateral are right angles, then the quadrilateral is a rectangle.
20. If a quadrilateral has four equal sides and four equal angles, then it is a square.
21. If two opposite angles of a quadrilateral both measure 90°, then the quadrilateral is a rectangle.
22. If an exterior angle of a triangle is acute, then the triangle is obtuse.
23. A pentagon with five equal sides is a regular pentagon.

Part C

24. Danielle conjectured that, if a point P is the vertex of two angles, $\angle APB$ and $\angle BPC$, then $\angle APC$ is obtuse. Draw a diagram for which the conjecture is true and one counterexample to show that the conjecture is false.
25. Pascal conjectured that, if line segments AB and BC have the same length, then B is the midpoint of the line segment AC. Draw a diagram for which the conjecture is true and one counterexample to show that the conjecture is false.
26. Sarita conjectured that, if the diagonals of a quadrilateral are perpendicular, then the quadrilateral cannot have all sides with different lengths. Draw diagrams to show whether her conjecture is true or false.
27. Ray conjectured that, if one diagonal of a quadrilateral bisects two interior angles, then the quadrilateral is a parallelogram. Draw diagrams to show whether his conjecture is true or false.