6.7: Exponential Growth and Growth Curves

There are many important applications of ***exponential growth*** in the fields of science and finance:

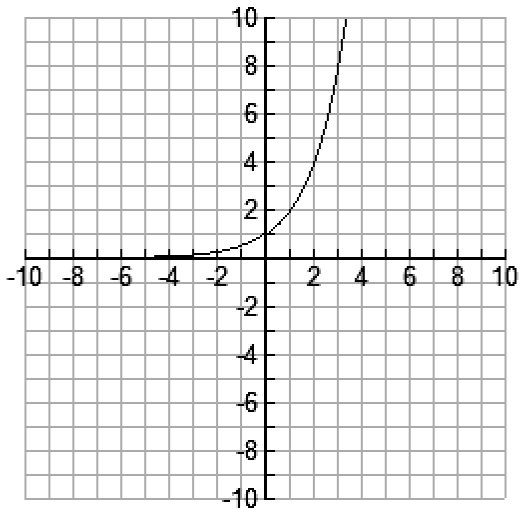
- describing the growth of a ***bacterial culture***
- calculation of ***compound interest***
- ***population growth***

Exponential Growth

Exponential Growth is modeled by an ***exponential function*** with the base $\boldsymbol{b}$, where $b > 1$.

Recall: A function $f(x) = a(b)^x$ for $b > 1$ is called an ***increasing*** function.

Graph of an increasing exponential function.



Notes:
The value of ***a*** is the y-intercept since…

if $\boldsymbol{x} = 0$, then
$$\begin{aligned} f(0) &= a(b)^0 \\ &= a(1) \\ &= a \end{aligned}$$

This can also be interpreted as the ***initial value*** for functions whose *domain* is $x \geq 0$.

The power of exponential growth:

Suppose you're given a choice between two "gifts". The first is $10 000 000 cash. The second is an account that starts with $2 and doubles each day for exactly 24 days. Which should you take?

In general, *Exponential Growth* can be modeled by the formula… $P(n) = P_o(1+r)^n$, where…

$P(n)$ → represents the **final** amount after the number of growth periods ***n***.

P_o → represents the **initial** amount

r → represents the **rate** of growth (represented as a **decimal** or a **fraction**)

Example of Exponential Growth Problem

A biologist grows a colony of bacteria in a Petri dish and finds that after **one day**, the quantity has increased by 50%. After **two days** it has increased again by 50% of the quantity noted after day one. The biologist is uncertain as to the exact **initial number of bacteria**. Determine an algebraic formula that can be used to describe the quantity after 10 days. After ***n*** days.

Solution: ***Let*** *P_o represent the initial number of bacteria*

Let's break this down into percentages…

The initial amount is represented as ....P_o

Write a formula for the given situation:

Population in a small town is predicted to grow at a rate of 8% per year. Currently there is 25 234 people in town. Determine a formula to calculate the population in 20 years.

Homework:

1. A population increases at a rate of 8.2%. What is the growth factor?

2. A population has a growth factor of 1.23. What is the rate of growth as a percent?

3. There are 1200 bacteria in a culture. Write an equation to represent the growth of the bacteria n days from now under each given condition:
   a) The population doubles every day.
   b) The population grows by a factor of 5 every day.
   c) The population increases at a rate of 1.05% per day.

4. Ontario's population in 1991 was approximately 10.1 million. The population has been increasing at a rate of 1.25% per year.
   a) Write an equation to represent the population of Ontario as a function of the number of years since 1991. Define your variables.
   b) Suppose the population continues to grow at this rate. Estimate the population in 2041.

5. A strain of bacteria doubles every half hour. Suppose there were 4000 bacteria when the timing began.
   a) Write an equation to model the population growth. Define your variables.
   b) How many bacteria would be present in 4 hours time?
   c) How many bacteria were present 2 hours ago?

6. A rare stamp was worth $65 in 1995. It was predicted to grow in value at a rate of 8% per year. At this growth rate, what would be the value of the stamp in 2020?

7. The table below shows the population of a town from 1993 to 1999.

Year	1993	1994	1995	1996	1997	1998	1999
Pop'n	500 000	525 000	551 250	578 813	607 754	638 142	670 049

   a) Does the data represent exponential growth? Explain.
   b) What is the growth factor (to 2 decimal places)? At what annual rate is the population increasing at?
   c) Write an equation to model the population, P, as a function of the number or years, n, since 1993.
   d) Predict the population of the town in 2010. What assumption are you making?

8. When a sheet of paper is folded in half, 2 layers are formed. If it is folded in half again, 4 layers are formed.
   a) Complete the table below for as many times as you can physically fold the paper.

# of folds	# of layers
0	1
…	…

   b) Write an equation to model the number of layers of paper as a function of the number of times the paper is folded.
   c) If the paper could be folded in half 30 times, how many layers would there be?