

Number Sense and Numeracy: Factors and Exponents

Example 1

$$2^4 = 2 \times 2 \times 2 \times 2$$

2 is called the power.

2 is called the base.

4 is called the exponent.

This is a power of 2.

expanded form

Example 2

Calculate $2^3 \times 2^2$.

Solution

$$2^3 \times 2^2 = 2 \times 2 \times 2 \times 2 \times 2$$

like bases

\ / = 32

Example 3

Evaluate $2^2 \times 3^2$.

Solution

$$2^2 \times 3^2 = 2 \times 2 \times 3 \times 3$$

\ / = 36

unlike bases

Practise

1. Write the value of each.

- | | | |
|-----------|-----------|-----------|
| (a) 2^2 | (b) 2^3 | (c) 2^4 |
| (d) 3^2 | (e) 3^3 | (f) 3^4 |
| (g) 4^2 | (h) 4^3 | (i) 5^3 |

2. Simplify.

- | | |
|----------------------|----------------------|
| (a) 3×2^2 | (b) 3×2^3 |
| (c) 2×4^2 | (d) 2×4^3 |
| (e) $2^2 \times 3^2$ | (f) $2^2 \times 3^3$ |
| (g) $3^2 \times 2^3$ | (h) $2^2 \times 4^2$ |
| (i) $2^3 \times 4^2$ | (j) $5^2 \times 3^2$ |

3. For each power, what is the base? the exponent?

- | | | |
|-----------|-----------|-----------|
| (a) 2^3 | (b) 3^2 | (c) 2^4 |
| (d) 3^4 | (e) 5^2 | |

4. Write each expression as a power.

- | |
|--|
| (a) $3 \times 3 \times 3 \times 3 \times 3$ |
| (b) $2 \times 2 \times 2 \times 2$ |
| (c) $5 \times 5 \times 5$ |
| (d) $4 \times 4 \times 4 \times 4 \times 4 \times 4$ |

5. Write in expanded form.

- | | | |
|--------------|-----------|--------------|
| (a) 5^2 | (b) 2^5 | (c) 6^4 |
| (d) x^3 | (e) y^4 | (f) $(2m)^3$ |
| (g) $(3n)^2$ | (h) 4^3 | (i) 3^4 |

6. Evaluate:

- | |
|----------------------------|
| (a) the third power of 2 |
| (b) the fourth power of 3 |
| (c) the second power of 5 |
| (d) the fifth power of 1 |
| (e) the sixth power of 0 |
| (f) the second power of 16 |

7. Write each number as a power of 10.

- | | |
|-------------|----------------|
| (a) 100 | (b) 1000 |
| (c) 100 000 | (d) 1 000 000 |
| (e) 10 | (f) 10 000 000 |

8. Write as a power of 2.

- | | |
|--------|---------|
| (a) 4 | (b) 16 |
| (c) 64 | (d) 256 |

9. Evaluate.

- | | |
|--------------------|----------------------|
| (a) $7^2 + 2^2$ | (b) $4^3 - 2^5$ |
| (c) 3×2^3 | (d) $3^2 \times 2^2$ |

10. Evaluate for $x = 2$.

- | | | |
|------------|----------------|-----------------|
| (a) $2x$ | (b) x^2 | (c) x^3 |
| (d) $3x^2$ | (e) $2x^2 + 3$ | (f) $x^3 - x^2$ |

11. Express each number as a product of two powers. (For instance, $36 = 2^2 \times 3^2$.)

- | | | |
|----------|--------|---------|
| (a) 100 | (b) 12 | (c) 108 |
| (d) 2500 | (e) 18 | (f) 72 |

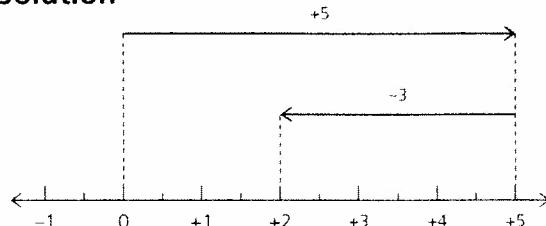
Number Sense and Numeracy: Integers—Addition and Subtraction

A number line can be used to show how to add integers.

Example 1

Find $(+5) + (-3)$.

Solution



$$(+5) + (-3) = 2 \quad \text{The positive sign is often not used.}$$

Practise

1. Find each sum.

- (a) $-3 + (-2)$ (b) $2 + (-3)$
(c) $-8 + (+8)$ (d) $-6 + (+4)$
(e) $-4 + (-5)$ (f) $2 + (-6)$

2. Add.

- (a) $\begin{array}{r} 5 \\ -7 \\ \hline \end{array}$ (b) $\begin{array}{r} -3 \\ -4 \\ \hline \end{array}$ (c) $\begin{array}{r} -9 \\ -2 \\ \hline \end{array}$
(d) $\begin{array}{r} -4 \\ -6 \\ \hline \end{array}$ (e) $\begin{array}{r} 7 \\ -7 \\ \hline \end{array}$ (f) $\begin{array}{r} 7 \\ -2 \\ \hline \end{array}$
(g) $\begin{array}{r} -8 \\ -4 \\ \hline \end{array}$ (h) $\begin{array}{r} -4 \\ -3 \\ \hline \end{array}$ (i) $\begin{array}{r} -5 \\ -5 \\ \hline \end{array}$

3. Find each difference.

- (a) $4 - (-3)$ (b) $-5 - (-2)$
(c) $5 - (-3)$ (d) $-4 - (-7)$
(e) $6 - (-6)$ (f) $4 - (4)$
(g) $-7 - (-3)$ (h) $-7 - (-9)$

4. Subtract.

- (a) $\begin{array}{r} -4 \\ -2 \\ \hline \end{array}$ (b) $\begin{array}{r} 5 \\ -3 \\ \hline \end{array}$ (c) $\begin{array}{r} -7 \\ -3 \\ \hline \end{array}$
(d) $\begin{array}{r} -5 \\ -5 \\ \hline \end{array}$ (e) $\begin{array}{r} -7 \\ -8 \\ \hline \end{array}$ (f) $\begin{array}{r} 7 \\ -3 \\ \hline \end{array}$

To subtract an integer you add its opposite.

Example 2

Subtract: (a) $5 - (-2)$ (b) $-8 - (+3)$

Solution

$$\begin{aligned} \text{(a)} \quad 5 - (-2) &= 5 + (+2) \\ &= 7 \end{aligned}$$

$$\begin{aligned} \text{(b)} \quad -8 - (+3) &= -8 + (-3) \\ &= -11 \end{aligned}$$

To subtract an integer, you add its opposite.

(g) $\begin{array}{r} -7 \\ -3 \\ \hline \end{array}$ (h) $\begin{array}{r} -7 \\ -3 \\ \hline \end{array}$

5. Simplify.

- (a) $3 - (-4)$ (b) $-7 + 2$
(c) $5 - 3$ (d) $3 - 5$
(e) $-4 - (-4)$ (f) $-4 - 4$
(g) $5 - (-3) + 4$ (h) $-4 - (-3) + 5$
(i) $-6 - 4 - 3$ (j) $-4 + 7 - 5$

6. Which choice would make each statement true: $>$, $<$, or $=$?

- (a) $-3 - 4 - 5 + 3 \blacksquare -4 - 3 - 1 - (-2)$
(b) $4 - 7 + 6 - 8 \blacksquare -3 - 5 - (-7) - 4$
(c) $9 - 6 - (-4) - 5 \blacksquare 5 - 13 - 7 - (-8)$
(d) $5 - 13 + 7 - 2 \blacksquare 4 - 5 - (-3) - 5$
(e) $7 - 3 - (-15) - 11 \blacksquare -7 - 3 - (-11) - 15$

7. In each row, which expression has the greatest value? the least value?

- (a) $-5 - 3 + 4, 4 - 3 - (-4), 5 - (-3) - 10$
(b) $4 - 3 - 1, -5 - (-2) + 4, -14 + 5 + 6$
(c) $9 - (-2) - 7, 5 - (-7) + (-9), -5 - 3 + 6$
(d) $-6 + 4 + 3 - 2, 4 - (-3) - 7, 5 - (-2) - 9$
(e) $-5 - 2 + 4, 3 - 12 + 2, -7 - (-2) + 1$

Number Sense and Numeracy: Integers—Multiplication and Division

Use a pattern to remember how to multiply or divide integers.

The + and – signs are shown here to remember the rules. In the examples and practise questions, +5 is written as 5.

$$\begin{array}{llll} (-5)(-2) = +10 & (-10) \div (-2) = +5 & (+5)(-2) = -10 & (-10) \div (+2) = -5 \\ (+5)(+2) = +10 & (+10) \div (+2) = +5 & (-5)(+2) = -10 & (+10) \div (-2) = -5 \\ \text{same signs} & \text{positive integer} & \text{different signs} & \text{negative integer} \end{array}$$

Example 1

Find $3(-2)$.

Solution

$$3(-2) = -6$$

Example 2

Calculate $(-2)^3$.

Solution

$$\begin{aligned} (-2)^3 &= (-2)(-2)(-2) \\ &= -8 \end{aligned}$$

Example 3

Find $(-18) \div (-3)$.

Solution

$$(-18) \div (-3) = 6$$

Practise

1. Find each product.

- (a) $(-3)(2)$ (b) $(-4)(-9)$
(c) $(4)(-3)$ (d) $(-7)(-3)$
(e) $(5)(4)$ (f) $(-2)(7)$

2. Simplify.

- (a) $-2(-7)$ (b) $-3(8)$ (c) $5(-7)$
(d) $-5(-7)$ (e) $-4(-9)$ (f) $-4(9)$

3. Find each quotient.

- (a) $-18 \div (-6)$ (b) $-24 \div 6$
(c) $51 \div (-17)$ (d) $-42 \div (-14)$
(e) $-18 \div (18)$ (f) $-24 \div (-6)$
(g) $60 \div (-12)$ (h) $-30 \div (-15)$

4. Simplify.

- (a) $\frac{-50}{5}$ (b) $\frac{-15}{-5}$ (c) $\frac{30}{-6}$
(d) $\frac{48}{-6}$ (e) $\frac{16}{-16}$ (f) $\frac{-16}{-8}$
(g) $\frac{18}{-9}$ (h) $\frac{-81}{27}$ (i) $\frac{-18}{-9}$

5. Evaluate.

- (a) $(-4)^2$ (b) $(-2)^4$ (c) $(-3)^3$

- (d) $(-5)^2$ (e) -5^2 (f) 4^3

- (g) -4^3 (h) $(-2)^5$ (i) $(-3)^2$

6. Simplify.

Example:

$$\begin{aligned} -3(-2)^4 &= -3(16) \quad \left[\begin{array}{l} \text{Calculate} \\ \text{powers first.} \end{array} \right] \\ &= -48 \end{aligned}$$

- (a) $-2(-3)^2$ (b) $4(-2)^3$
(c) $5(-3)^3$ (d) $(-3)^2(-2)^2$
(e) $-3^2(-2)^3$ (f) $(5)^2(-2)^2$
(g) $-5^2(-3)$ (h) $(-5)^2(-3)$

7. Calculate.

- (a) $(-6)^2 \div (-3)$ (b) $-6^3 \div (-3)$
(c) $-3^4 \div (-3)^2$ (d) $(-4)^3 \div (-2)^3$
(e) $6^2 \div (-3)^2$ (f) $-4^2 \div (-2)^3$
(g) $2(-4)^2 \div (-8)$ (h) $-8 \div [(-2)(4)]$
(i) $-8(-3) \div (-2)^2$

8. Calculate.

- (a) $(5^2 \div 5) \times (7^2 \div 7)$
(b) $(4^3 \div 2^2) \div (2 \times 2^2)$
(c) $(-4^3 \times 3) \times (3^2 \div 3)$

Number Sense and Numeracy: Order of Operations

When calculating expressions, we follow the order of operations:

Order of Operations

1. Simplify brackets.
2. Then simplify powers.
3. Multiply and divide.
4. Then add and subtract.

Example 1

Simplify

$$-3(2 - 4) - (-2 + 4)$$

Solution

$$\begin{aligned} & -3(2 - 4) - (-2 + 4) \\ &= -3(-2) - (2) \\ &= 6 - 2 \\ &= 4 \end{aligned}$$

Example 2

Add $(-3)^2$ to the product of -2 and 4 .

Solution

$$\begin{aligned} & (-2)(4) + (-3)^2 \\ &= (-2)(4) + 9 \\ &= -8 + 9 \\ &= 1 \end{aligned}$$

Practise

1. Simplify. Use the order of operations.

- (a) $5 - (3 - 4)$
- (b) $(5 - 7) - (3 - 4)$
- (c) $-3(-4) - (5 - 7)$
- (d) $(3)(2) - (3 + 5)$
- (e) $-(5 - 9) - (-2)(2)$
- (f) $(4 - 3) - 2(3 - 4)$
- (g) $4(-2) - (-8 + 4)$

2. Simplify.

- (a) $2(-3)^2 - 4(-2)$
- (b) $-4(-2)^3 - 3(-4)^2$
- (c) $(-3 - 2)^2 - (2 + 4)^2$
- (d) $3(-2 + 4)^3 - 2(-4 + 1)^2$
- (e) $2(-1 - 3)^2 - (1 + 3)^2$
- (f) $5(-2)^2 - 3(-1 - 2)^3$
- (g) $3(-1 - 2) - (5 - 7)^3$
- (h) $5(-2 + 1)^3 - (-3 - 2)^2$
- (i) $4(1 - 3)^3 - (4 - 7)^2$
- (j) $-2(1 - 4)^3 - 3(1 - 5)^2$

3. Match each expression with the corresponding sentence.

- (a) Find the sum of -3 , -8 , and -2 .
- (b) -8 is added to the product of -3 and -2 .

- (c) Subtract -8 from the product of -3 and -2 .

- (d) Divide -8 by -2 and add -3 .

- (e) Decrease the quotient of -8 and -2 by -3 .

- i. $-3(-2) + (-8)$

- ii. $-8 \div (-2) + (-3)$

- iii. $-8 \div (-2) - (-3)$

- iv. $-8 + (-3) + (-2)$

- v. $(-3)(-2) - (-8)$

4. Increase the sum of -3 and 5 by -6 .

5. Divide the sum of 7 and -16 by -3 .

6. By how much is the sum of -8 and 6 more than -4 ?

7. How much less is the sum of -8 and -4 than the product of 4 and -2 ?

8. Divide -4^2 by $(-2)^2$.

9. Add -3^2 and 4^2 to the product of 6 and -2 .

10. Increase the product of -7 and -3 by -18 .

11. By how much is $(-3 + 5)^2$ more than $-(-3 + 5)^2$?

12. Divide the sum of -8 , -11 , 7 , and -3 by 5 .

13. Divide the sum of -4^2 and 2^4 by -7^2 .

$$\text{Add: } \frac{7}{10} + \frac{2}{15}$$

Solution

$$\begin{aligned}\frac{7}{10} + \frac{2}{15} &= \frac{21}{30} + \frac{4}{30} & \text{Subtract: } 2\frac{1}{4} - \frac{1}{2} \\&= \frac{25}{30} & \text{Find a common denominator.} \\&= \frac{5}{6} & \text{Express in lowest terms.}\end{aligned}$$

Practise

1. Write the missing information to form equivalent fractions.

$$\begin{array}{llll}(\mathbf{a}) \frac{1}{3} = \frac{\blacksquare}{18} & (\mathbf{b}) \frac{\blacksquare}{36} = \frac{1}{9} & (\mathbf{c}) \frac{\blacksquare}{28} = \frac{4}{7} \\(\mathbf{d}) \frac{1}{5} = \frac{7}{\blacksquare} & (\mathbf{e}) \frac{3}{8} = \frac{15}{\blacksquare} & (\mathbf{f}) \frac{18}{\blacksquare} = \frac{2}{9} \\(\mathbf{g}) \frac{1}{\blacksquare} = \frac{9}{36} & (\mathbf{h}) \frac{3}{\blacksquare} = \frac{15}{55} & (\mathbf{i}) \frac{5}{35} = \frac{\blacksquare}{7}\end{array}$$

2. Add.

$$\begin{array}{llll}(\mathbf{a}) \frac{1}{7} + \frac{3}{7} & (\mathbf{b}) \frac{2}{9} + \frac{5}{9} & (\mathbf{c}) \frac{3}{8} + \frac{1}{8} \\(\mathbf{d}) \frac{1}{3} + \frac{1}{9} & (\mathbf{e}) \frac{1}{3} + \frac{1}{6} & (\mathbf{f}) \frac{1}{3} + \frac{5}{12}\end{array}$$

3. Subtract.

$$\begin{array}{llll}(\mathbf{a}) \frac{5}{9} - \frac{1}{9} & (\mathbf{b}) \frac{14}{15} - \frac{7}{15} & (\mathbf{c}) \frac{7}{15} - \frac{2}{5} \\(\mathbf{d}) \frac{5}{6} - \frac{3}{8} & (\mathbf{e}) \frac{3}{4} - \frac{1}{6} & (\mathbf{f}) \frac{1}{3} - \frac{1}{6}\end{array}$$

4. Add.

$$\begin{array}{llll}(\mathbf{a}) 1\frac{1}{6} + 2\frac{1}{6} & (\mathbf{b}) 2\frac{3}{10} + 1\frac{3}{10} & \\(\mathbf{c}) 3\frac{3}{4} + 1\frac{3}{4} & (\mathbf{d}) 1\frac{3}{4} + 2\frac{5}{12} & \\(\mathbf{e}) 2\frac{1}{6} + 1\frac{5}{6} & (\mathbf{f}) 4\frac{2}{5} + 3\frac{1}{5} &\end{array}$$

7. Divide.

$$\begin{array}{llll}(\mathbf{a}) \frac{3}{7} \div \frac{4}{5} & (\mathbf{b}) \frac{2}{11} \div \frac{3}{5} & \\(\mathbf{c}) \frac{3}{4} \div \frac{7}{8} & (\mathbf{d}) \frac{5}{8} \div \frac{13}{16} & \\(\mathbf{e}) 2 \div \frac{2}{3} & (\mathbf{f}) 4 \div \frac{8}{9} & \\(\mathbf{g}) \frac{3}{4} \div 9 & (\mathbf{h}) \frac{5}{7} \div 10 &\end{array}$$

8. Arrange the fractions in order from least to greatest in value.

$$\begin{array}{llll}(\mathbf{a}) \frac{3}{4}, \frac{5}{8}, \frac{1}{2} & (\mathbf{b}) \frac{7}{8}, \frac{3}{4}, \frac{13}{16} & \\(\mathbf{c}) \frac{3}{5}, \frac{9}{10}, \frac{3}{4} & (\mathbf{d}) \frac{5}{6}, \frac{8}{9}, \frac{2}{3} &\end{array}$$

$$\text{Divide } 1\frac{2}{3} \div \frac{3}{10}$$

Solution

$$\begin{aligned}\frac{7}{10} + \frac{2}{15} &= 2\frac{1}{4} - \frac{1}{2} & \text{Multiply: } \frac{3}{5} \times 1\frac{1}{9} \\&= \frac{25}{30} & \text{Find a common denominator.} \\&= \frac{5}{6} & \text{Express in lowest terms.}\end{aligned}$$

Solution

$$\begin{aligned}\frac{3}{5} \times 1\frac{1}{9} &= \frac{3}{5} \times \frac{10}{9} \\&= \frac{15}{45} \times \frac{10}{9} \\&= \frac{2}{3} & \text{Express in lowest terms.} \\&& \text{Then multiply.}\end{aligned}$$

$$\begin{aligned}\frac{1\frac{2}{3}}{1\frac{3}{10}} &= \frac{\frac{5}{3}}{\frac{10}{3}} \\&= \frac{5}{10} & \text{Multiply by the reciprocal.} \\&= \frac{5}{9} & = 5\frac{5}{9}\end{aligned}$$

Solution

$$\begin{aligned}\frac{4\frac{7}{16}}{2\frac{3}{16}} &= \frac{\frac{63}{16}}{\frac{35}{16}} \\&= 2 & \text{Divide by the reciprocal.}\end{aligned}$$

Solution

$$\begin{aligned}\frac{3\frac{5}{9}}{1\frac{7}{9}} &= \frac{\frac{30}{9}}{\frac{16}{9}} \\&= \frac{3}{16} & \text{Divide by the reciprocal.}\end{aligned}$$

Review of Essential Skills and Knowledge – PART I