6.8: Exponential Decay and Decay Curves

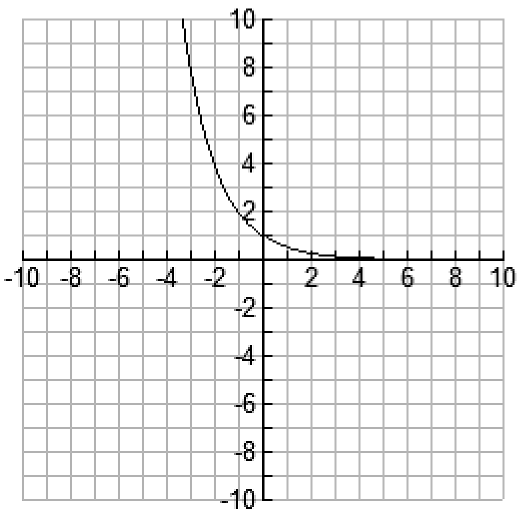
There is a very common application of ***exponential decay*** in the fields of science, and that is the deterioration of a ***radioactive material***.

Exponential Decay

Exponential Decay is modeled by an exponential function with the base ***b*** where $0 < b < 1$.

Recall: A function $f(x) = a(b)^x$ where $0 < b < 1$ is called a **decreasing** function.

Graph of a decreasing exponential function.



Notes:
The value of ***a*** is the y-intercept since…

if ***x*** = 0, then

This can also be interpreted as the ***initial value*** for functions whose *domain* is $x \geq 0$.

Example of Exponential Decay (Half – Life)

Half-life is the period of time required for half of the atoms of specified quantity of matter to disintegrate.

The half-life of radon is 4 days. Find the expression for the mass of radon remaining from a sample of 15 grams after 16 days; After ***x*** days.

Solution:

After 4 days the weight of radon is-----------

After 8 days the weight of radon is ----------

<u>**After 12 days**</u> the weight of radon is ---------

<u>**After 16 days**</u> the weight of radon is ---------

<u>**After x days**</u> the weight of radon is ----------

In general, *Exponential Decay* (**half-life**) is defined by the equation

$$n = n_0\left(\frac{1}{2}\right)^{\frac{t}{h}}$$

where $\boldsymbol{n_0}$

$\boldsymbol{n}$

$\boldsymbol{h}$

Note: $\boldsymbol{t}$ and $\boldsymbol{h}$ must be measured using the same units.

<u>**Example**</u>: (see pg 436 Example 3) How much Cobalt-60 will there be after 42 months?

Similar to the *growth* formula seen in the previous lesson, we can develop a *decay* formula…

In general, $P(n) = P_o(1-r)^n$ can be used to describe ***ANY*** **exponential decay** situation, where…

P(n) → represents the **final** amount after the number of decay periods ***n***.

P_o → represents the **initial** amount

r → represents the **rate** of decay

Example:

A scuba diver knows that light intensity **decreases** by 2% for each metre below the surface. His user manual give the function $I(n) = I_o(0.98)^n$ where I_o is the initial amount of light at the surface and ***n*** is the depth in metres.

a) In the formula above, how was the common ratio of 0.98 calculated based on the information given?

b) He needs a minimum of 60% of light to operate without the aid of additional light sources. Is he able to take pictures at a depth of 23m without the aid of additional light sources?

The **initial amount of light** I_o = **____________**

c) Determine the ***rate of decay*** using the general formula.

Homework:

1. A population decreases at a rate of 4.5%. What is the decay factor?

2. A population has a decay factor of 0.91. What is the percent decrease in the population?

3. There are currently 1000 deer in a provincial park. Write an equation to represent the number of deer, y, in the park, x years from now under each condition:
   a) The population increases at a rate of 3% a year.
   b) The population decreases at a rate of 3% a year.

4. A radioactive isotope has a half-life of 5 years. A laboratory has a 24-g sample of the isotope.
   a) Write an equation to represent the mass of the sample, y grams, left after x half-lives.
   b) How many half-lives will have elapsed in 125 years?
   c) How much of the sample is left after 125 years?

5. When light passes through ice, its intensity is reduced by 4% for every 1 cm thickness of ice.
   a) Write an equation to express the percent of light, P, that penetrates x centimetres of ice.
   b) What percent of light penetrates a sheet of ice 4.5 cm thick?

6. Blue jeans fade with repeated washing. Suppose a pair of jeans loses 2% of its colour after each wash. Write an equation to model this situation and then use the equation to determine how much of the original colour is left after 50 washings?

7. A laboratory has a 500 gram sample of nitrogen-13. This substance has a half-life of approximately 10 minutes. Write an equation to model this situation and then use the equation to determine the amount of the sample remaining after 1.5 hours.

8. Verify the following table represents exponential decay and then write an equation that could be used to model the situation.

Years	0	1	2	3	4
Whale pop'n	1000	950	902	857	814