5.8: Maxima/Minima Problems from Standard Form

For many problems, it is desirable to find the largest or smallest outcome. For example, a manufacturer may want to know when the largest profit is obtained in a process or when the smallest amount of raw material is used. By setting the problems up using a quadratic function, the largest, or smallest value is found by completing the square and finding the vertex.

Example #1: Two numbers have a difference pf 8. Find the numbers if their product is a minimum.

Example #2: The sum of a number and 3 times another number is 18. Find the number if the product is a maximum.

Example #3: A rectangular lot is bounded on one side by a river and on the other 3 sides by 80m of fencing. Find the dimensions of the lot with the largest area.

Example #4: A bullet is fired vertically at 80m/s. Its height above the ground is given by $h = 80t - 5t^2$. Find the maximum height of the bullet and when it occurs.

Example #5: Sam hits a pop-up. The height h in meters of the baseball is given by $h = 1.2 + 20t - 5t^2$, where t is in seconds. What is the maximum height of the ball? If it is caught at the same height that it was hit, how long was the ball in the air?

Example #6: Lemon Motors monthly revenue from car sales is given by $R = -20x^2 + 400x + 48\,000$, where x represents the markup above factory price in multiples of \$20. Find the markup needed for maximum revenue.

Practice Problems:

Demonstrate that the vertex obtained by completing the square and by factoring are identical. Round answers to 3 decimal places.

a) $2x^2 + 13x + 6$
b) $6x^2 - 21x - 12$
c) $-2x^2 - 7x - 3$
d) $-4x^2 + 15x - 9$
e) $2x^2 + 17x + 35$