## 4.2: The Characteristics of a Function

A relation is a set of ordered pairs. These can be represented in various ways.

## Examples of Relations:

a) $\{(1,2),(5,3),(9,4),(7,1)\}$
...as a mapping diagram

b) $\{(1,3),(4,2),(3,2),(6,5)\}$ $\ldots$ as a table of values

| $\mathbf{x}$ | $\mathbf{y}$ |
| :---: | :---: |
| 1 | 3 |
| 3 | 2 |
| 4 | 2 |
| 6 | 5 |

c) $\{(1,4),(3,2),(5,4),(3,1)\}$
$\ldots$ as a scatter plot


The DOMAIN is the set of first elements of the ordered pairs (the set of distinct $\mathbf{x}$ values)
The RANGE is the set of second elements of the ordered pairs (the set of distinct $\mathbf{y}$ values) (see pg 7 of the textbook for alternate definitions)

For each example above we can write the Domain and Range using SET NOTATION...

| Example a) has...Domain $=\{$ | $\}$ | and | Range $=\{$ | $\}$ |
| :--- | :--- | :--- | :--- | :--- |
| Example b) has...Domain $=\{$ | $\}$ | and | Range $=\{$ | $\}$ |
| Example c) has...Domain $=\{$ | $\}$ | and | Range $=\{$ | $\}$ |

A FUNCTION is a relation in which each value in the domain corresponds to exactly ONE element of range. It can also be thought of as a rule that associates each $x$ value with only ONE $y$-value.
Note: More than one $x$-value can correspond to the same $y$-value.

A relation is NOT a function if one $x$ value has 2 different $y$-values associated with it.
In the examples above, example a) and b) are functions.

Example c) is not a function since the $x$-value 3 is associated with two $y$-values... $y=1$ and $y=2$
To visualize this, complete a MAPPING diagram for example c)


One other thing... since we are dealing with functions, but not all equations represent functions, we're going to use a special type of notation when using functions. It's called FUNCTION NOTATION!

| Equation | Function Notation |
| :---: | :---: |
| $y=3 x+1$ | $f(x)=3 x+1$ |
| $y=3 t^{2}-2 t+1$ | $g(t)=3 t^{2}-2 t+1$ |
| $y=x^{3}-2 x^{2}+1$ | $h(x)=x^{3}-2 x^{2}+1$ |
| $y=t^{2}-21$ | $v(t)=t^{2}-21$ |



Using function notation is similar to using equations involving $\boldsymbol{x}$ and $\boldsymbol{y}$ values. To find a $\boldsymbol{y}$-value given an $\boldsymbol{x}$-value simply requires substitution. Thus, we can write ordered pairs $(x, f(x))$ which are the same as $(x, y)$.

Example 1: Find $f(2)$ if $f(x)=x^{2}-2 x+1$. (Here, we are looking for the " $y$-value" when $x=2$.)

Example 2: Given the function $g(x)=4 x-5$, find...
a) $2 g(-1)$
b) $g(a+2)$

